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Default- and call-adjusted duration for corporate bonds

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Abstract

Call and default can potentially alter the timing and amounts of promised cashflows for callable, corporate bonds. While prior research has indicated the theoretical importance of adjusting Macaulay duration for the impacts of default and call, the question of their relative impact remains a matter of debate [The High Yield Debt Market, Dow Jones Irwin, New York, 1990, p. 18; J. Finan. 53 (1998) 2225]. We develop a theoretical analysis incorporating both default and call effects on duration and test its implications employing a previously unexplored data base of Canadian, investment grade, corporate bond indices containing an unusual provision making it possible to identify callable and noncallable indices.

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1. Introduction

Two events can potentially alter the timing and amounts of promised cashflows for callable, corporate bonds: Call and default. While prior research has indicated

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the theoretical importance of adjusting Macaulay duration for the impacts of default and call, the question of their relative impact remains a matter of debate. Fons (1990) concludes that default risk explains his finding that risk-adjusted duration is significantly lower than Macaulay duration for his sample of US corporate bond indices. In contrast, Duffee (1998) points out that US corporate bond indices are comprised of callable bonds and cautions that the impact of callability likely confounds tests seeking to isolate the impact of default risk.

The present paper advances our understanding by developing a theoretical analysis extending Fons incorporating both default and call effects on duration. We go on to test the implications of our framework employing a previously unexplored data base of Canadian, investment grade, corporate bond indices containing an unusual provision making it possible to separate callable and noncallable corporate bonds. This unique feature allows us to measure separately the effects on duration of callability and default risk.

Our findings challenge those of Fons but are consistent with Duffee's warning – call effects generally dominate those of default risk for callable indices in our sample. For portfolio managers holding callable corporate bonds, the message is that the possibility of call outweighs that of default in impacting on duration.

The remainder of this paper is organized as follows; Section 2 provides a short review of the literature related to the default and call adjustment of duration. Section 3 explores further the theoretical ramifications of callability and default risks. Section 4 describes our data and outlines the testable hypotheses. The testing methodology we use is outlined in Section 5. Section 6 reports our results. Finally, Section 7 provides a summary and our conclusions.

2. The risk and call adjustment for corporate bonds: Theory and evidence

Fons (1990) presents a theoretical framework for the adjustment of duration for the risk of default. He concludes that risk-adjusted duration should be lower than its Macaulay counterpart. This conclusion depends heavily on his assertion that a bond's credit quality is negatively correlated with the level of the riskless term structure. Fons' empirical evidence demonstrates that, during the 1980–1988 period, the risk-adjusted duration of corporate bond indices of all ratings was significantly lower than their Macaulay duration. In his study, Fons does not consider callability an important factor in measuring the bond's price sensitivity to changes in the riskless rate. However, many of the corporate bonds included in the indices he uses are callable and, as we show below, callability is perhaps an even more important adjustment factor for highly rated corporate bonds relative to the adjustment for default risk.

In contrast with Fons' (1990) study which draws a comparison between values obtained for the risk-adjusted duration with the riskless yield as underlying reference and the Macaulay duration with the risky yield as the underlying reference, Babbel, Merrill, and Panning (BMP) (1997) conduct a different comparison by using the riskless yield for both duration measures. Thus, they compare interest sensitivities of risk-prone and risk-less bonds and isolate the effects of default risk. BMP utilize option-pricing technology to calculate risk-adjusted duration and compare it to the duration of a matched synthetic Treasury bond. They find that risk-adjusted duration is lower than riskless duration for all levels of yield spreads, with the difference monotonically increasing with yield spreads. However, it is unclear whether BMP's results are statistically robust. Also, the implications of their results are less important to bond portfolio managers who use Macaulay duration with the risky yield as the underlying reference, or duration measures of its family, to estimate the sensitivity of the value of a risky bond to changes in the riskless term structure.

Fooladi, Roberts, and Skinner (FRS) (1997) derive a risk-adjusted duration measure for corporate bonds while considering both risk aversion of investors and a delay period in recovery in the default process (a feature introduced in a different form by Bierwag and Kaufman (1988)). FRS show that incorporating the delay period, especially for zero-coupon bonds with a material risk of default, may result in a risk-adjusted duration greater than the bond's time to maturity. This result is attributed to the positive probability that the issuer of a zero-coupon bond will default at maturity and bondholders will receive any recovered amount only after a certain waiting period (delay period).

Jacoby (2003) also includes risk aversion and delay in recovery, and finds a similar result. Jacoby's risk-adjusted duration may also be greater than the bond's maturity. However, it will always be greater than the Fisher–Weil duration, or Macaulay duration when a flat term structure is assumed, due to the delay period. Like FRS, Jacoby assumes independence between the probability of default and the riskless term structure.

Chance (1990) draws on Merton's (1974) option-pricing model for the valuation of risky pure discount bonds. Using a result obtained by Garman (1985) for the sensitivity of an option's value to changes in interest rates, Chance expresses the duration of the discount bond as the weighted average of the duration of a riskless discount bond and that of a put option written on the firm's assets. Like Merton, Chance assumes that the firm's assets follow a diffusion process independent of the short riskless rate process. Therefore, the duration of the firm's assets must be zero.

Nawalkha (1996) extends Chance's model by allowing dependence between the two processes, implying a nonzero duration of the firm's assets. Unlike Chance's duration, Nawalkha's duration is not restricted to be lower than the maturity of the risky discount bonds. In our context, since the Macaulay duration of a pure discount bond is its time to maturity, Nawalkha's model implies that the risk-adjusted duration of a pure discount corporate bond can be either lower than, equal to, or higher than its Macaulay duration. [2]

Acharya and Carpenter (2000) develop a model for the valuation of callable defaultable bonds. In their paper, both interest rates and firm value are stochastic,

[2] Other papers applying option-pricing technology to duration include Dunetz and Mahoney (1988) and Brooks and Attinger (1992).

and the call and default decisions are endogonized. With respect to interest-rate sensitivity, as in other models applying option-pricing technology, [3] their model implies that default risk alone reduces the bond's duration. Acharya and Carpenter also show that call risk, when isolated from the risk of default, will also shorten bond duration. When considering both default and call risks together, they demonstrate that as the probability of default increases, the impact of the call option on the bond duration diminishes. A higher default risk provides an incentive to the issuer to wait longer before calling the bond, and results in a higher effective duration for the bond.

Kihn (1994) explores default and call risks for corporate bonds. He pays special attention to the case where both credit quality and interest rates decline. For low-grade bonds the credit deterioration will depress bond prices to a level low enough to discourage issuers from exercising the call provision. Kihn provides empirical support for his hypothesis by analyzing return volatility of high-grade and low-grade bonds. His results support the theoretical implications of Acharya and Carpenter's bond pricing model.

Our work builds on Fons (1990) emphasizing the importance of the error, and the financial damage to follow, of using Macaulay duration rather than the duration adjusted for both risks. Following Fons, we directly estimate the relationship between risk-adjusted duration and its Macaulay counterpart. To this end, we fit regression models offered by Fons with econometric techniques we believe to be superior to his and test hypotheses drawn from the theoretical framework derived below. In contrast to Fons' analysis which ignores call risk, we offer a unified theoretical analysis focusing on both default and call risks. Finally, while both Fons and Kihn use corporate bond data which are subject to both default and call risks, we employ a unique database of Canadian corporate bond indices which allows us to isolate the effect of default risk on duration analysis and compare it with the impact of callability.

3. Default and callability adjustment for duration

This section presents a unified analysis for default and call adjustments to duration. We do not attempt to derive a closed form solution but use the elasticity definition of duration. This framework is found to be quite useful in forming hypotheses with respect to the relationship of default- and call-adjusted duration to its Macaulay counterpart. In order to make the analysis more manageable, we assume a flat term

[3] For example Longstaff and Schwartz (1995) demonstrate that under risk-neutral valuation, the market value of the firm's assets is expected to grow at the riskless rate. This implies that following an increase in the riskless rate, the risk-neutral probability of default will decline, and a lower risk-neutral credit spread should be demanded. This means that default risk in such models reduces the bond's interest-rate sensitivity.

structure for both the risky rate at y, and the riskless rate at r. [4] Thus, the risk-adjusted duration of a noncallable bond is given by $D_{\mathrm{a}} = -(1/V)(\partial V/\partial r)$. [5] It can be shown that the following relationship holds: [6]

$$\frac{D_{\mathrm{a}}}{D_{\mathrm{m}}} = \frac{\partial y}{\partial r}, \tag{1}$$

where $D_{\mathrm{m}} = -(1/V)(\partial V/\partial y)$ is the bond's ordinary Macaulay duration (with the risky yield as the reference yield), and y the bond's continuously compounded (risky) yield to maturity.

Defining $g(\cdot)$ as the function relating the risky yield (y) to the riskless rate (r), the probability of default (p), and other parameters, Fons (1990) shows that

$$\frac{\partial y}{\partial r} = \frac{\partial g}{\partial r} + \frac{\partial g}{\partial p}\frac{\partial p}{\partial r}. \tag{2}$$

He assumes that the first term on the right-hand side of Eq. (2) is positive. The term $\partial g/\partial p$ is positive as well since investors will demand higher yields when default risk is higher.

The sign of $\partial p/\partial r$ depends on the correlation between default risk and the level of riskless rates. Fons adopts the view that these two variables are negatively correlated. However, this issue is an open empirical question. [7] The term $\partial_g/\partial r$, represents shifts in the bond's yield attributable to shifts in the Treasury yield with no reference to resultant changes in the bond's credit risk. Therefore, $\partial_g/\partial r$ must be unity. Eqs. (1) and (2) imply that allowing for a negative sign for $\partial p/\partial r$ may result in the unadjusted Macaulay duration (D_{m}) being greater than risk-adjusted duration (D_{a}).

As previously discussed, Fons provides empirical evidence showing that D_{m} is indeed greater than D_{a}. However, Duffee (1998) shows that the use of such indices is problematic since many of the bonds in these indices are callable. He provides empirical evidence for a weak relationship between credit spreads on noncallable investment-grade bonds and Treasury yields, implying $\partial p/\partial r = 0$. However, for callable bonds this relationship becomes strong and negative. Following a given increase in the riskless rate, the yield of a corporate bond will increase similarly, and its price will decline. This will reduce the probability of a call and cause an increase in the

[4] Assuming a flat term structure may lead to some loss of generality. However, the purpose of making this assumption is to derive a general expression which allows hypothesizing with respect to the impact of call and default risks on Macaulay duration. In addition, this assumption is consistent with empirical techniques of measuring the price elasticity of bonds. Note that alternatively one can assume that y and r are short rates. This assumption allows the analysis below to hold under all multiple factor term-structure models, and it implies that duration is a partial risk measure with respect to the short rate (see Chambers et al., 1988; Nawalkha, 1999). We thank the anonymous referee for suggesting this alternative assumption.

[5] In deriving this expression and expressions to follow, we assume continuously compounded rates of return. Thus, the following expression is the continuously compounding case which corresponds to the discrete case derived by Fons (1990).

[6] The derivation is available on request.

[7] For a discussion of the correlation between default risk and the level of riskless rates, see Nawalkha (1996) and Jacoby (2003).

bond price and an offsetting decline in its risky yield. Thus, the bond's yield will rise less than the Treasury yield, resulting in a negative relationship between its yield spread and the riskless yield.

Duffee's results indicate that, following a given change in the Treasury yield, the yield on a noncallable bond will tend to change by a similar amount while the yield on a callable bond will change by a smaller amount. Thus, letting $\partial p/\partial r = 0$, causes the price elasticity (duration) of a callable bond with respect to changes in the Treasury yield ($D_{\mathrm{a}}^{\mathrm{c}}$) to be smaller than with respect to its own yield ($D_{\mathrm{m}}^{\mathrm{c}}$), as we show below. Duffee's empirical evidence implies that, for noncallable investment grade bonds, these two measures should be close. To formalize these observations, we alter Fons' (1990) framework, and define the yield to maturity of a risky callable bond (y^{c}) as the sum of the yield to maturity of a corresponding riskless bond (r) and a yield spread. The yield spread is a function of the bond's time to maturity (T), the risky coupon rate (c), the coupon rate of a corresponding riskless bond (c_f), the conditional probability of default (p), the probability of a call (ϕ), the call price (E), the call-protected period (T_{CP}), and a measure for investors' risk preferences (δ): [8]

$$y^{\mathrm{c}} = r + f(T, c, c_f, p, \phi, E, T_{\mathrm{CP}}, \delta). \tag{3}$$

In order to analyze the relationship between y^{c} and r, we must first note that the probabilities of default and call are related to the riskless rate (among other parameters). Therefore, we can write

$$p = h(r, \ldots) \quad \text{and} \quad \phi = k(r, \ldots).$$

Given the functional form defined for the risky yield (y^{c}) and relating it to the riskless yield (r) in Eq. (3), the following relationship must hold between Macaulay duration ($D_{\mathrm{m}}^{\mathrm{c}}$) and the default- and call-adjusted duration ($D_{\mathrm{a}}^{\mathrm{c}}$) for callable bonds:

$$D_{\mathrm{a}}^{\mathrm{c}} = D_{\mathrm{m}}^{\mathrm{c}} \frac{\partial y^{\mathrm{c}}}{\partial r}. \tag{4}$$

The default- and call-adjusted duration of the bond is equal to the product of its call-adjusted Macaulay duration and the sensitivity of its default- and call-adjusted yield with respect to changes in the riskless yield. With the functional forms stated for the probability of default (p) and the probability of a call (ϕ), we get [9]

$$\frac{\partial y^{\mathrm{c}}}{\partial r} = 1 + \frac{\partial f}{\partial p}\frac{\partial p}{\partial r} + \frac{\partial f}{\partial \phi}\frac{\partial \phi}{\partial r}. \tag{5}$$

Comparing Eq. (5) with the relationship derived by Fons (1990) (Eq. (2) above), unlike Fons, we chose not to ignore call risk. Thus, we see that for callable bonds we now have an extra term, the last term on the right-hand side of Eq. (5).

To make Eq. (5) more manageable, we have to take a closer look at the signs of the last two terms on its right-hand side. For the second term, which refers to default

[8] Fons does not consider the parameters related to the call provision.

[9] At this point we ignore the higher order cross terms which give rise to the interaction between call and default risks. Kihn (1994) demonstrates that this term is nontrivial only for low-grade bonds in times of depressed business cycles and declining interest rates. We revisit this issue below when we discuss our results.

risk, Fons (1990) notes that the term $\partial f/\partial p$ is positive since investors will demand higher yields when default risk is higher. The sign of $\partial f/\partial r$ depends on the correlation between default risk and the level of riskless rates.

Next, we examine the last term on the right-hand side of Eq. (5), which refers to call risk. The sign of $\partial f/\partial \phi$ will always be positive, since a higher chance of a call implies a higher call premium demanded by bondholders, and a higher risk-adjusted yield. On the other hand, $\partial \phi/\partial r$ will always be negative since higher Treasury yields imply a lower chance of the issuer exercising the option provided by the call provision and this will cause bondholders to demand a lower call premium. Therefore, unlike the case of the noncallable bond, the magnitude of the risk- and call-adjusted duration depends not only on how default risk changes with shifts in the riskless rate ($\partial p/\partial r$), but also on the added term related to callability $((\partial g/\partial \phi)(\partial \phi/\partial r))$. In general, $\partial y^{c}/\partial r$ will be lower than $\partial y/\partial r$ of a corresponding noncallable bond due to the negative sign of $(\partial g/\partial \phi)(\partial \phi/\partial r)$.

Together with Eqs. (4) and (5), all this leads to the following relationship between the default- and call-adjusted duration and its Macaulay counterpart:

$$\frac{D_{\mathrm{a}}^{\mathrm{c}}}{D_{\mathrm{m}}^{\mathrm{c}}} = 1 + \underbrace{\underset{(+)\,(?)}{\frac{\partial f \partial p}{\partial p \partial r}}}_{(?)} + \underbrace{\underset{(+)\,(-)}{\frac{\partial f \partial \phi}{\partial \phi\, \partial r}}}_{(-)}. \tag{6}$$

It is useful to rewrite Eq. (6) as

$$\frac{D_{\mathrm{a}}^{\mathrm{c}}}{D_{\mathrm{m}}^{\mathrm{c}}} = 1 + \underbrace{\text{Default Term}}_{(?)} + \underbrace{\text{Callability Term}}_{(-)}.$$

Our four hypotheses address risk- and call-adjusted duration vs. Macaulay duration. Given the negative sign of the callability term for the callable indices, we hypothesize the following:

H1: When the default term $((\partial f/\partial p)(\partial p/\partial r))$ is negative as well, the default- and call-adjusted duration should be smaller than Macaulay duration for callable bonds ($D_{\mathrm{a}}^{\mathrm{c}}/D_{\mathrm{m}}^{\mathrm{c}} < 1$).

H2: When the default term $((\partial f/\partial p)(\partial p/\partial r))$ is positive, this relationship depends on the magnitude of the positive default term relative to the magnitude of the negative callability term.

For the noncallable indices, for which the callability term drops out of Eq. (6), we hypothesize:

H3: When the default term $((\partial f/\partial p)(\partial p/\partial r))$ is negative, the default-adjusted duration should be lower than or equal to Macaulay duration for noncallable bonds ($D_{\mathrm{a}}/D_{\mathrm{m}} < 1$).

H4: When the default term $((\partial f/\partial p)(\partial p/\partial r))$ is positive, in the absence of the callability term, we expect that $D_{\mathrm{a}} > D_{\mathrm{m}}$.

Since we cannot estimate the magnitude of $(\partial f/\partial p)(\mathrm{d}p/\mathrm{d}r)$, then for high-grade bonds, for which the default term can be trivial, one can expect $D_{\mathrm{a}} \cong D_{\mathrm{m}}$. It follows that an empirical finding that the expressions in H3 and H4 are not significantly different from equality would be consistent with our theory.

4. Empirical implications of default risk and callability for duration analysis

In this section we estimate the relationship between default- and call-adjusted duration and Macaulay duration for indices of corporate bonds which mainly include callable bonds.

4.1. Description of the data

Our sample consists of month-end price and yield-to-maturity data from Scotia Capital Markets (SCM) investment grade Canadian corporate bond indices for the 12-year period from January 1986 to December 1997, reported by Statistics Canada (CANSIM). [10] The SCM corporate bond indices include all investment grade Canadian publicly traded corporate bonds payable in Canadian dollars with more than 1 year to maturity. A call provision is attached to most of these bonds. SCM assigns bonds into a single index according to their credit ratings supplied by the Canadian Bond Rating Service (CBRS) or the Dominion Bond Rating Service (DBRS). When a split rating occurs, SCM designates a rating according to its best judgment based on credit analysis. The four different investment grade rating categories are AAA, AA, A, and BBB. Starting in April 1993, SCM discontinued the AAA index as no bonds fit this category.

The SCM corporate universe index is an aggregation over credit ratings and term-to-maturity buckets. SCM further decomposes the aggregate into an array of four credit-rating sectors by three term buckets, each representing a maturity group. For example, the short-term *i*-rated corporate bond index, $i =$ AAA, AA, A, BBB, includes all corporate bonds in the *i*-rated Universe index with remaining terms to maturity of 1–5 years. The mid-term and long-term corporate bond indices represent bonds with remaining terms to maturity ranging from over 5 to 10 years, and greater than 10 years, respectively. [11] In brief, our sample consists of time series of

[10] The following description of the SCM indexes is based on Hatch and Robinson (1988),and on discussions and correspondence with Robert Bose, Associate and Fixed Income Analyst, Global Fixed Income, Scotia Capital Markets.

[11] Data for the AAA indices of all maturities are available only until March 1993. Historically, SCM imposed constraints on the range of coupon rates permitted for corporate bonds to be included in the indexes. These constraints were designed to eliminate the coupon level effect from the yield spread of the included bonds over the yield of Government of Canada bonds, so that this spread is as close as possible to the true credit spread. At present, corporate bonds have relatively lower coupon rates, and these constraints are no longer necessary. SCM has decided to drop these restrictions following the objective of better representing the Canadian secondary bond market. A table describing the data is available from the authors.

16 different corporate bond indices; for each of the 4 rating groups we have one Universe portfolio and three sub-portfolios representing the different maturity ranges.

Month-end prices and yields to maturity for each index (and other analytics), are calculated by SCM using market capitalization weights. The capitalization weights represent the outstanding value of each individual bond issue, net of "uninvestible" bonds relative to the total index net outstanding market value. [12] To create market capitalization values for every holding, these "investible" amounts outstanding are used in conjunction with daily market closing prices and interest income accrual. Weights are revised daily to reflect shifts in the composition of the index, as well as changes in value arising from price movements and accrued interest. These changes are a result of the changing maturity of bonds, bond retirements, new issues, and shifts in the bonds' credit rating.

All individual bond yields and prices are from the SCM Bond Desk as quoted daily at 4:00 p.m. EST. To calculate prices, SCM traders use a matrix-pricing algorithm that calculates a yield for each bond based on a benchmark Government of Canada yield plus a spread. The spreads used in the algorithm are based on credit analysis, and dynamically revised with the arrival of any new information reflecting changes in market conditions. According to SCM, spread revisions normally lead changes in ratings provided by the rating agencies. The SCM indices are the prevailing indices for the Canadian bond market (both government and corporate) and are widely accepted by most practitioners in the Canadian fixed-income market.

According to Warga (1991), institutional bond data, such as SCM trader-quoted prices, are both more comprehensive and more accurate than exchange data because over-the-counter bond traders deal in a considerably larger volume of bond trades than do exchanges. Moreover, Warga asserts that month-end institutional data are more reliable than daily quotes due to the more extensive profit/loss calculations made at month end.

For each term-to-maturity bucket of corporate bonds, we have to identify a Government of Canada index with a similar term to maturity. We assume that, on average, the term to maturity of each index is close to the average maturity of its maturity bucket matching a constant maturity, Government of Canada index to every corporate index. For example, for the short-term indices with terms of individual bonds ranging from 1 to 5 years, we assume an average portfolio maturity of 3 years. We choose the 3-, 7-, and 10-year constant maturity Government of Canada bond indices to correspond to the different corporate bond maturity buckets respectively. [13] For the universe indices of the different ratings, which contain corporate bonds of all maturities, we assign the 7-year constant maturity Government of Canada bond index. Yield data for the three constant maturity Government of Canada bond indices are obtained from CANSIM.

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[12] Outstanding par values are adjusted for Bank of Canada holdings, holdings of the Caisse de Depot et Placement du Quebec, and strip market activity.

[13] As of December 31, 1997, the weighted average maturity for corporate bonds in the short-term indices ranged from 3.1 to 3.4 years, for the mid-term indices it ranged from 7.7 to 8.7 years, and for the long-term indices the weighted average term to maturity ranged from 14.5 to 21.1 years.

4.2. Data characteristics

In Eq. (6), the sign of $\partial p/\partial r$ is open to empirical determination and here we present a methodology to determine its sign by taking a closer look at a proxy, $\Delta p/\Delta r$. [14] To determine Δr, we calculate the average annual first difference in the yield on the 7-year constant maturity Government of Canada index during each period (similar results were obtained for the 3- and 10-year indices). Since we are more interested in the sign of $\partial p/\partial r$ rather than its magnitude, we require an index that proxies the direction the credit quality of bonds takes in different cycles of the economy. Because SCM's bond indices are stratified by credit rating, a natural proxy for the sign of Δp is the number of rating upgrades relative to the number of downgrades within a given period. [15] For example, suppose that for a given period, bonds in an index with credit rating i, $i =$ AAA, AA, A, BBB, enjoy significantly more upgrades than downgrades, we argue that for this index, credit quality improves on average. Therefore, the sign of Δp during this period is negative, implying a reduction in the probability of default. [16]

The actual credit rating changes we use are based on Canadian Bond Rating Service (1986–1997). Two possible problems may arise from the use of these reports. First, as discussed above, SCM uses both CBRS and DBRS ratings in the allocation of individual bonds into an index that is homogeneous by credit-rating class. When a split rating occurs, and SCM agrees with the DBRS rating, our reliance on the CBRS rating may be problematic. However, comparing the ratings of the two agencies for a portfolio of 32 Canadian corporate bonds, Kaplan (1998) shows that there is no evidence that the rating differences are systematic. Since we deal with bond indices (portfolios), it is reasonable to assume that split rating differences are random, and that, on average, the two rating agencies agree on the credit quality of the bond portfolio. Moreover, Kaplan finds that most rating differences are of only one notch while the SCM indices do not distinguish rating categories based on single notches but rather distinguish based on groups of three notches each.

Second, rating changes tend to lag several months after the actual shift in credit quality of the issuer (Weinstein, 1977). To avoid this problem, we focus on rating changes which occur during a relatively long period – 4 years. Thus, a few months'

[14] Our proxy, $\Delta p/\Delta r$ is a total derivative; that is, Δp is the total change in the probability of default followed by a small change in the riskless rate. Moving from theory to practice, it is necessary to assume that the partial derivative captures most of the effect of the total derivative. Following this assumption, we determine the sign of $\partial p/\partial r$ by identifying it with the sign of $\Delta p/\Delta r$.

[15] Nawalkha (1996) shows that the relationship between default risk and interest rates ($\partial p/\partial r$) is sector specific. Since in our study we look at the impact of call and default risks on the duration of bond indices (portfolios) representing the different credit-rating groups, we need to determine $\partial p/\partial r$ for each such portfolio, instead of focusing on the different business sectors.

[16] For AAA-rated bonds, with the highest credit quality, Δp is more likely to be negative. Note however, that since the AAA rating category has three notches, Δp can also be positive. This implies that Δp for AAA-rated bonds is truncated. We thank the anonymous referee for highlighting this point.

lag in rating changes will affect only the beginning and the end of the 4-year period, and so the effect of these changes is minimized. [17]

The resultant signs of Δr and Δp indicate that our 12-year sample is characterized by three equal sub-periods, each representing a different economic cycle in the Canadian corporate bond universe. [18] During the January 1986–December 1989 period, the number of corporate bond rating upgrades (92) clearly exceeded the number of downgrades (54), which implies a negative sign for Δp. The yield on the 7 year, constant maturity Government of Canada index increased during the same 4-year period, implying a positive sign for Δr; the average annual first difference is +0.07%. Thus, when combining the ameliorated credit quality of the Canadian corporate bond universe during this sub-period with the increase in riskless rates, one can conclude that the sign of $\partial p/\partial r$ is negative for these bonds. The January 1990–December 1993 period coinciding with the recession of the early 1990's is characterized by a downshift in the riskless term structure ($\Delta r < 0$) combined with a deteriorated credit quality of the Canadian corporate bond universe ($\Delta p > 0$). Finally, during the January 1994–December 1997 period, the riskless term structure continued to shift downwards ($\Delta r < 0$), while the Canadian economy entered the booming period of the mid-1990's, which triggered the credit-quality amelioration of the Canadian corporate bond universe during this period ($\Delta p < 0$). Thus, the sign of $\partial p/\partial r$ switched to positive for most Canadian corporate bonds during this sub-period.

4.3. The doomsday call provision

Kaplan (1998) describes a major structural change related to the call provision attached to Canadian corporate bonds during our sample period. Until 1986 a substantial number of Canadian corporate debt issues had a standard call provision with a "not for financial advantage" clause designed to allow the issuer to call the bond but not to replace it with a cheaper source of debt. The implications of such a clause, if enforceable, are that bondholders will rarely face a financial loss related to a call provision attached to the bond. In 1986, a court decision made it clear to Canadian

[17] Note that in the above methodology for the determination of the sign of $\Delta p/\Delta r$, there is no event-for-an-event matching of changes in bond rating in response to changes in the proxy for the riskless rate. We use the number of rating upgrades relative to the number of downgrades within a given year to proxy Δp, and Δr is the annual first difference in the yield on the 7-year constant maturity Government of Canada index. To get a closer match between shifts in the riskless rate and rating changes, we also look at weekly changes in the yield on the 7-year constant maturity Government of Canada index, and characterize a year as having positive (negative) Δr when there are more (less) upward shifts in rates than downward shifts. We find that for all years the sign of Δr is identical under the two measures. The results of this alternative methodology are available on request.

[18] A table showing the number of upgrades and downgrades in each period is available from the authors. A Chow test for structural breaks conducted in the regression analysis below strongly supports this division of our 12-year sample to three distinct 4-year sub-periods.

investors that the "not for financial advantage" clause was unenforceable. [19] This motivated the replacement of the standard call provision with the Canada call (or doomsday call). A doomsday call provision sets the call price at the maximum of par value or the value of the bond calculated based on the yield on a matching Government of Canada bond plus a spread. Because Canadian bonds are usually traded with yield spreads much wider than those set in the call provision, the exercise of the doomsday call provision will almost never cause financial damage to the bondholders.

The existence of the doomsday call has strong implications for our study because it makes bondholders practically immune to the impact of changes in the riskless term structure on the value of the call provision. While for the standard call, lower riskless rates implied a higher probability of the issuer calling the bond, with the doomsday call, the effect of lower riskless rates is neutralized by the call price floating upwards, and we get: $\partial\phi/\partial r = 0$. Hence, for bonds with a doomsday call, the callability term in Eq. (6) disappears, leaving default as the only factor that may affect the relationship between the risk-adjusted duration and Macaulay duration. Thus, the existence of the Canada call provides an effective tool for the isolation of the effect of the default-risk adjustment and its importance.

According to Kaplan, starting in 1987, almost all Canadian corporate bonds with maturities greater than 5 years were issued with a doomsday call. Since our long-term indices include only bonds with more than 10 years to maturity, the vast majority during the last sub-period will be those issued after 01:1987, carrying the doomsday call rather than the standard call provision. [20] Our mid-term indices, representing the 5- to 10-year bucket, will include a mix of bonds carrying both types of call provisions. Assuming that, on average, the rate of arrival of new issues into the mid-term indices is uniform, it is reasonable to assert that most of the bonds included during the last sub-period carry the doomsday call. For the short-term indices, corresponding to the 1- to 5-year term-bucket, it is more likely that most bonds are those originally issued prior to 1987 with the standard call provision. [21]

In summary, applying Eq. (6) to our long-term and mid-term bond indices during the 01:1994–12:1997 period, we expect the callability term to disappear due to the doomsday call.

4.4. Testable hypotheses

At this point it is useful to summarize the application of hypotheses H1–H4 (presented above) to our data. For the 1986–1989 period, the AA, A, and BBB indices

[19] In June of 1986, Dofasco announced its decision to call outstanding callable debentures issued in 1982. Manufacturers Life Insurance Co., a major bondholder with holdings worth $40 million Canadian, decided to pursue litigation, claiming that Dofasco violated the "not for financial advantage", clause attached to the call provision of its bonds. However, the court sided with Dofasco's claim that the issue was not called for financial advantage.

[20] Since the makeup of the SCM indices is proprietary, we must resort to inference here.

[21] Since most Canadian corporate bonds are issued with 10–20 years to maturity, it is more likely that the number of newly issued short-term bonds is dominated by the number of seasoned bonds included in the short-term indices.

(all with $(\partial f/\partial p)(\partial p/\partial r) < 0$) fall into hypothesis H1, while the AAA index (with $(\partial f/\partial p)(\partial p/\partial r) > 0$) falls into hypothesis H2. For the 1990–1993 period, all indices have a negative $(\partial f/\partial p)(\partial p/\partial r)$, and hence, they fall into hypothesis H1. Finally, for the 1994–1997 period, for the short-term callable indices, the AA index (with $(\partial f/\partial p)(\partial p/\partial r) < 0$) falls into hypothesis H1, while the A and BBB indices (with $(\partial f/\partial p)(\partial p/\partial r) > 0$) fall into hypothesis H2. For the noncallable medium-term and long-term indices during this period, the AA index (with $(\partial f/\partial p)(\partial p/\partial r) < 0$) falls into hypothesis H3, while the A and BBB indices (with $(\partial f/\partial p)(\partial p/\partial r) > 0$) fall into hypothesis H4.

For the last sub-period we argue above that the vast majority of bonds in the long-term bond indices carry the doomsday call. So we expect the effect of the callability term on the required call adjustment for duration to diminish as we move from the beginning of our sample period to its end. As a result, for the long-term bond indices, the ratio D_a^c/D_m^c will tend to be higher as we progress in time from the first sub-period to the last sub-period, due to the inclusion of more and more new bonds carrying a doomsday call.

5. Test methodology

Our estimation technique follows a methodology offered by Fons (1990). We estimate the empirical relationship between risk-adjusted duration and its Macaulay counterpart (D_a^c/D_m^c) for every index i_{Mk}, $i_{Mk} = \mathrm{AAA}_{Mk}, \mathrm{AA}_{Mk}, \mathrm{A}_{Mk}, \mathrm{BBB}_{Mk}$, in a maturity bucket M, $M =$ Universe, short-term, mid-term, long-term, during each sub-period k, $k =$ 01:1986–12:1989, 01:1990–12:1993, 01:1994–12:1997, with the following regression model: [22]

$$y^c_{i_{Mk}t_k} = \gamma_{i_{Mk}} + \delta_{i_{Mk}} r_{t_k} + u_{i_{Mk}t_k}, \quad t_k = 1, \ldots, 48, \tag{7}$$

where $y^c_{i_{Mk}t_k}$ is the continuously compounded yield on bond index i_{Mk} at time t_k (calculated as: ln[1 + observed yield on bond index i_{Mk}]); r_{t_k} the continuously compounded yield on the Government of Canada constant maturity index corresponding to the maturity group of bond index i_{Mk} at time t_k (as described in the previous section); $\gamma_{i_{Mk}}$ the intercept; $\delta_{i_{Mk}}$ the slope; $u_{i_{Mk}t_k}$ the error term; t_k the relative month in period k, $k =$ 01:1986–12:1989, 01:1990–12:1993, 01:1994–12:1997. [23]

[22] We also employ an alternative methodology, which entails independently estimating D_a^c and D_m^c using regression analysis, and then obtaining an estimate for D_a^c/D_m^c by taking the ratio of the two estimates. An independent ordinary least squares (OLS) estimation of the two duration measures is statistically inefficient due to the high correlation between the regression residuals. To avoid this problem, we jointly estimate D_a^c and D_m^c using the three-stage least squares (3SLS) method (see Gallant, 1977). In general, the results obtained when using the 3SLS estimation are similar to the results of estimating regression model (7). Given the limitations of scope and space, the 3SLS results are available on request.

[23] Data for AAA-rated bonds are available only until March 1993; thus for the 01:1994–12:1997 period we have only 39 monthly observations, and $t_{01:1994-12:1997} = 1, \ldots, 39$.

The formulation offered by regression model (7) is desirable since, for bond index i_{Mk}, the theoretical relationship between the default- and call-adjusted duration and Macaulay duration is given by $D_{\mathrm{a}}^{\mathrm{c}}(i_{Mk})/D_{\mathrm{m}}^{\mathrm{c}}(i_{Mk}) = \partial y_{i_{Mk}t_k}^{\mathrm{c}}/\partial r_{t_k}$, and the slope coefficient of the above regression model is given by $\partial y_{i_{Mk}t_k}^{\mathrm{c}}/\partial r_{t_k} = \delta_{i_{Mk}}$. Thus, $\hat{\delta}_{i_{Mk}}$, the value of the slope estimator, directly estimates $D_{\mathrm{a}}^{\mathrm{c}}(i_{Mk})/D_{\mathrm{m}}^{\mathrm{c}}(i_{Mk})$.

The US bond indices used by Fons are equivalent to our universe indices which include corporate bonds of all maturities for each credit-rating category. His sample consists of the 116 months from December 1979 to July 1988. During such a long period, the levels of the different duration measures of such portfolios may have fluctuated significantly due to their continuously changing composition. The constrained nature of the SCM indices in terms of their stratification by maturity mitigates this problem, and poses a lesser threat to the stationarity assumption for the regression slope estimators in our study.

Both data sets share the problem of the autoregressive nature of the residuals produced by applying the OLS procedure to regression model (7). When residuals follow an autoregressive process, one should expect the OLS procedure to yield estimates of regression parameters with high t-values, combined with high adjusted R^2's and low Durbin–Watson values for the residuals. Fons makes the somewhat restrictive assumption that the residuals follow a first-order autoregressive process (AR(1)). Therefore, he uses the Cochrane and Orcutt (1949) iterative procedure to adjust for first-order serial correlation. However, the OLS residuals may actually follow a higher order autoregressive process. In this case, results produced by the first-order Cochrane–Orcutt adjustment may be meaningless with respect to the true relationship in the data. To guard against this problem. we use Akaike's information criteria (AIC) to select the degree, q, of the autoregressive process which best fits the OLS residuals produced by the above regression models. When estimating regression model (7), we find that, although in most cases OLS residuals follow an AR(1) process, a higher autoregressive (up to AR(6)) order is present in a nontrivial number of cases.

Given the changing order of the autoregressive process followed by the OLS residuals, and given the relatively small number of observations in each sub-period sample used, we apply the more flexible Yule–Walker method (sometimes called the two-step full transform procedure). This estimation method is adaptable to the order of the required serial correlation adjustment, and unlike the Cochrane–Orcutt, it does not eliminate observations. [24]

6. Regression results

The results of estimating regression model (7) for the full period, 01:1986–12:1997, are in Table 1. For every SCM corporate bond index the table shows the estimates of the regression parameters with their t-values; the adjusted R^2; the order of the auto-

[24] Gallant and Goebell (1976) describe the Yule–Walker method in detail.

Table 1
Estimation of the relationship between the default- and call-adjusted duration and its corresponding Macaulay duration for the entire sample period: 01:1986–12:1989

Index	Slope	Constant	Adj. R^2	q	DW(q)	Iter	SE(δ)	$t(\delta < 1)$	Chow_1	Chow_2	Obs
Universe											
AAA	0.7817	0.0265	0.92	2	1.7415	3	0.0257	8.4942	32.9043	N/A	87
	(30.389)	(11.181)							(0.0001)		
AA	0.8244	0.0196	0.85	3	1.8500	5	0.0290	6.0552	16.7759	98.3102	144
	(28.462)	(5.620)							(0.0001)	(0.0001)	
A	0.8348	0.0208	0.86	3	1.8182	4	0.0281	5.8790	7.7851	103.1966	144
	(29.664)	(5.694)							(0.0006)	(0.0001)	
BBB	0.7293	0.0341	0.62	1	2.1054	3	0.0468	5.7842	5.47	120.1732	144
	(15.57)	(6.236)							(0.0052)	(0.0001)	
Short-term											
AAA	0.8271	0.0206	0.92	2	1.9172	1	0.0268	6.4515	5.4172	N/A	87
	(30.911)	(8.663)							(0.0061)		
AA	0.7890	0.0223	0.85	2	1.9300	5	0.0284	7.4296	11.7553	88.1796	144
	(27.826)	(7.110)							(0.0001)	(0.0001)	
A	0.8487	0.0189	0.84	3	2.0059	3	0.0309	4.8964	8.1603	127.4156	144
	(27.434)	(4.995)							(0.0004)	(0.0001)	
BBB	0.7339	0.0333	0.69	2	1.9732	3	0.0404	6.5866	5.6064	114.3885	144
	(18.186)	(6.857)							(0.0045)	(0.0001)	
Mid-term											
AAA	0.8783	0.0167	0.93	2	1.9151	1	0.0249	4.8876	20.0094	N/A	87
	(35.233)	(7.405)							(0.0001)		
AA	0.8821	0.0154	0.87	3	1.8405	6	0.0287	4.1080	11.9628	50.9939	144
	(30.708)	(5.492)							(0.0001)	(0.0001)	
A	0.8664	0.0187	0.84	3	1.7897	4	0.0323	4.1362	6.4567	94.4773	144
	(26.834)	(4.928)							(0.0021)	(0.0001)	
BBB	0.7938	0.0301	0.54	1	2.0773	2	0.0600	3.4367	5.2002	85.5929	144
	(13.226)	(5.341)							(0.0066)	(0.0001)	
Long-term											
AAA	0.7623	0.0307	0.87	1	1.8786	2	0.0308	7.7175	28.4354	N/A	87
	(24.712)	(10.878)							(0.0001)		
AA	0.8327	0.0220	0.87	3	2.0176	4	0.0274	6.1058	8.8136	111.8601	144
	(30.371)	(7.995)							(0.0002)	(0.0001)	
A	0.8128	0.0253	0.87	3	1.9913	4	0.0267	7.0112	4.9032	149.4417	144
	(30.454)	(8.819)							(0.0087)	(0.0001)	
BBB	0.7279	0.0412	0.42	1	2.1143	2	0.0720	3.7792	13.43	87.8292	144
	(10.114)	(6.339)							(0.0001)	(0.0001)	

$$y^{c}_{i_M t} = \gamma_{i_M} + \delta_{i_M} r_t + \Psi_{i_M t},$$

where $y^{c}_{i_M t}$ is the continuously compounded yield on index i_M at time t, $M =$ universe, short-term (1–5 years), mid-term (5–10 years), long-term (over 10 years), and $i_M = \text{AAA}_M, \text{AA}_M, \text{A}_M, \text{BBB}_M$. r_i is the continuously compounded yield on a Government of Canada constant-maturity index corresponding to the maturity group of index i_M, δ_{i_M} is the slope coefficient which represents the relationship between the default- and call-adjusted duration of index i_M and its Macaulay duration. γ_{i_M} is the intercept, and $\Psi_{i_M t}$ is the error term. t-values are in parentheses. In addition to the estimates of the regression parameters and the adjusted R^2, the table reports the following: q gives the degree of the autoregressive process as determined by the AIC, DW(q) reports the Durbin–Watson statistic adjusted for the selected

(*continued on next page*)

Table 1 (*continued*)
autoregressive order (q), Iter is the number of iterations performed by the Yule–Walker procedure, SE(δ) is the standard error of the slope estimator, and $t(\delta < 1)$ is the t-statistic used to test the null hypothesis that the slope coefficient is unity. Obs is the number of monthly observations (data for all AAA indices are available only until March 1993). Chow_1 and Chow_2 report the results of the Chow test for structural breaks between the three periods. The p-values for the Chow tests are in parentheses.

regressive process followed by residuals from an OLS estimation of regression model (7) as selected by the AIC; a Durbin–Watson statistic adjusted for the selected autoregressive order; the number of iterations performed by the Yule–Walker procedure before convergence of the estimated parameter; the standard error of the slope estimator; and, finally, the one-tailed t-statistic for the null hypothesis that the slope is unity. If the null hypothesis is rejected, we may conclude that the ratio $(D_{\mathrm{a}}^{\mathrm{c}}/D_{\mathrm{m}}^{\mathrm{c}})$ is lower than one, implying that empirical default- and call-adjusted duration is lower than empirical Macaulay duration for the underlying index. For the pooled data, we conduct a Chow test for structural breaks between the three sub-periods. Table 1 further reports values for the Chow tests, Chow_1 and Chow_2, and their respective p-values.

Contrasting the results obtained for the pooled data with those within each sample period, reveals that ignoring period-specific characteristics related to callability and credit quality, can lead to confusion. The regression slope estimators for the pooled sample are always below one at high levels of statistical significance as shown in Table 1. Thus, observing the results for the pooled data only, reinforces Fons' conclusion, that empirical default- and call-adjusted duration is lower than its Macaulay counterpart. However, stratifying the data, leads to the opposite conclusion in some cases.

Above, we argue for stratifying the data into three equal sub-periods, based on demonstrated period-specific credit-quality and callability characteristics. The Chow tests for structural breaks strongly support our stratification with the relevant Chow statistics significant at the 1% level for indices of all credit-rating and maturity groups, suggesting that a closer examination of this relationship within each sub-period is necessary.

To conduct that examination we re-run the regressions for the four different rating groups decomposed into four different term buckets (including the Universe index) and three sub periods, a total of 44 regressions. [25] The slope estimators of all regression models are highly significantly different from zero. Adjusted R^2's are mostly higher than 0.8, which implies that regression model (7) fits the data well. The combination of high t-values, high adjusted R^2's, and Durbin–Watson values close to 2.0 implies that the Yule–Walker method handles the problem of autoregressive residuals with efficiency while maintaining the model's goodness-of-fit.

In Table 2 we summarize hypotheses H1–H4, together with the results of the Yule–Walker procedure, with respect to the magnitude of the $D_{\mathrm{a}}^{\mathrm{c}}/D_{\mathrm{m}}^{\mathrm{c}}$ ratio. To summarize our results for hypotheses H1–H4, we focus only on the results for the

[25] The full regression output is available on request.

Table 2
Results of estimation of the relationship between the default- and call-adjusted duration and its corresponding Macaulay duration by term-bucket for each of the sub-periods: 01:1986–12:1989, 01:1990–12:1993, and 01:1994–12:1997

Rating	1986–1989				1990–1993				1994–1997				
	H1–H2	ST	MT	LT	H1–H2	ST	MT	LT	H1–H2	ST	H3–H4	MT	LT
AAA	?	0.7363*	0.8035*	0.7291*	<1	0.8599*	0.9178*	0.7640*	N/A	N/A	N/A	N/A	N/A
AA	<1	0.6903*	0.7917*	0.7098*	<1	0.8909*	0.9597	0.8795*	<1	0.8547*	$\leqslant 1$	1.0124	0.9619
A	<1	0.7036*	0.7449*	0.7303*	<1	0.9098*	0.9418	0.8346*	?	0.7907*	$\geqslant 1$	0.8670**	0.8972**
BBB	<1	0.6693*	0.8106*	0.6833*	<1	0.7260*	0.5325*	0.3551*	?	0.6936*	$\geqslant 1$	1.0004	0.9417

Hypotheses H1–H2 refer to indices defined as callable, hypotheses H3–H4 refer to indices defined as noncallable (all long-term and mid-term indices during the last sub-period). Along with the hypothesized magnitude of the estimated ratio, we report the Yule–Walker estimator for the short-term (ST), mid-term (MT), and long-term (LT) indices of every rating category.

*Significantly lower than one at the 1% level.

**Significantly lower than one at the 2.5% level.

term-bucket indices, since Universe indices will tend to average out term-bucket specific characteristics. These hypotheses for the relationship between the default- and call-adjusted duration and Macaulay duration described by Eq. (6) gain support from the data for 29 out of the 33 cases we analyzed.

6.1. Interpretation of regression results

We see that in most cases the call adjustment is more important than the default-risk adjustment. For example, we conclude that $D_a^c/D_m^c < 1$ for the AAA-rated indices of all term buckets during the first sub-period. However, AAA-rated bonds clearly suffered from credit-quality deterioration during that period, and $(\partial f/\partial p) \times (\partial p/\partial r)$ was therefore determined to be positive. This, together with Eq. (6), means that the callability term is greater in absolute terms than the default term, and we can comfortably conclude that callability causes a greater distortion for Macaulay duration when compared to the distortion caused by default risk for AAA bonds during the first sub-period. A similar conclusion can be made for short-term A-rated bonds during the last period.

Fons (1990) concludes that default risk is the reason for his results wherein the empirical risk-adjusted duration is always significantly below the empirical Macaulay duration of US corporate bond indices of all ratings. Our results clearly indicate that Fons' results likely proxy for the impact of callability rather than measure the impact of default. Duffee (1998) warns against interpreting results of yield-spread studies based on US corporate bond indices, which include primarily callable bonds. We extend this warning to the use of such bond indices for researching the impact of default risk on the duration of corporate bonds.

Our data also support the view that, *ceteris paribus*, with the inclusion in the long-term indices of more and more new bonds carrying the economically noncallable doomsday call, the magnitude of the callability term will diminish over time. This means that we expect the ratio D_a^c/D_m^c for these indices to increase over time. The results in Table 2 agree with this hypothesis. The slope coefficient for long-term indices of all ratings is monotonically increasing as we move from the first sub-period to the last sub-period. For the long-term BBB index, the slope estimator is still generally increasing with time – however, not monotonically. [26] Finally, during the 01:1990–12:1993 recessionary period, which is also characterized by declining interest rates, we attach a negative sign for $\partial p/\partial r$ for all indices. Kihn (1994) shows that during such periods, prices of low-grade bonds will tend to decrease, due to deteriorated credit quality, to a level low enough to eliminate the incentive to call these bonds which arises due to the lower interest rates. Comparing our results for the second sub-period with the results for the other two periods, it appears that the call ad-

[26] To test the statistical significance of these comparisons, we apply a *t*-test, proposed by Edwards (1984, pp. 91–93), for comparing the magnitude of the slope coefficient in regression model (7) across time. With the exception of the AAA index, for which data are available only until March 1993, the comparisons of the relative slope magnitudes are all statistically significant at the 1% level. Given the limitations of scope and space, the results produced by this *t*-test are available on request.

justment is less important during the second sub-period. The slope estimator of D_a^c/D_m^c for any given index during the 01:1990–12:1993 period is higher than that estimated for the same index during the first sub-period. Considering the callable short-term bond indices during the last sub-period, we find again that the slope estimator for the corresponding short-term index during the 01:1990–12:1993 period is larger. This is with the exception of the BBB mid-term and long-term bond indices, for which the slope estimator during the second sub-period is lower than those estimated for the other two sub-periods. [27]

Thus, the results for our investment-grade bonds support Kihn's conclusion that during recessionary periods, combined with declining interest rates, the impact of call risk is less important. This also provides support for Acharya and Carpenter's (2000) bond pricing model. Their model implies that a higher risk of default provides an incentive to the issuer to wait longer before calling the bond, and results in a higher effective duration for the bond.

6.2. Four special cases

The only four cases where the implications of Eq. (6) are violated are:

(1) During the 01:1990–12:1993 period, for the mid-term AA- and A-rated indices we expect $D_a^c/D_m^c < 1$, since we estimate $(\partial f/\partial p)(\partial p/\partial r)$ to be negative and expect the callability term to be negative as well. However, this hypothesis is not supported by the data.

(2) During the 01:1994–12:1997 period, for the noncallable long-term and mid-term A- rated indices we expect $D_a/D_m \geqslant 1$, since we expect the callability term to be zero and estimate $(\partial f/\partial p)(\partial p/\partial r)$ to be positive. The regression analysis provides a slope estimator for which we reject the hypothesis that it is unity and conclude that $D_a/D_m < 1$.

The implication of (1) is that, although both $(\partial f/\partial p)(\partial p/\partial r)$ and the callability term are negative for the mid-term AA and A indices, for this period we cannot find any distortion in the Macaulay duration caused either by callability or by default. In light of these results, one might conclude that both effects were trivial for those indices during the second period. We believe that this is highly unlikely in view of the collapse of the Canadian real-estate sector during the 1991–1993 period.

Canadian Bond Rating Service (1997) shows that, excluding defaults prompted by the real-estate crisis, the default rates for A++, A+, and A-rated bonds are 0.00%, 0.64%, and 1.58%, respectively, for the 1973–1996 period. However, when the 1992 defaults of Bramalea and Olympia and York and the 1993 defaults of Trizec and Royal Trust are accounted for, the historical default rates for the 1973–1996 period spike to 3.10%, 2.59%, and 3.71% for the highest three ratings, respectively. The

[27] Again, to test the statistical significance of these comparisons, we apply Edwards' (1984) t-test for comparing the magnitude of the slope coefficient in regression model (7) across time. The discussed comparisons are all statistically significant at the 5% level. Results are available on request.

default rates for these ratings during the shorter 1990–1993 sub-period must have been much higher, and were probably more like those observed for high-yield bonds. This suggests that $(\partial f/\partial p)(\partial p/\partial r)$ was negative during our second sub-period.

We argue that the skyrocketing default rates for AA- and A-rated bonds (equivalent to the A+ and A CBRS ratings) during the second sub-period resulted in depressed market prices for these bonds. For a bond carrying a standard call option, depressed prices imply that the call option will be deep out of the money, with a trivial probability of the call being exercised for the financial advantage of the issuer. Thus, depressed bond prices for AA- and A-rated bonds, make them economically noncallable during the second sub-period, and therefore we can expect to get: $\partial\phi/\partial r = 0$. Hence, for bonds included in the mid-term indices of these ratings, the callability term in Eq. (6) disappears, leaving default as the only factor that may affect the relationship between the risk-adjusted duration and Macaulay duration. [28] Furthermore, the effect of depressed prices is reflected in the results for AA- and A-rated indices of all term-buckets. Close observation will reveal that the slope estimators of these ratings within each term-bucket are much higher than those estimated for the AAA- and BBB-rated indices. However, this pattern does not exist in the first and last sub-periods.

To strengthen this point, we obtain the dollar amount of bonds outstanding for real-estate-related companies at the year-end prior to their default. Since the SCM indices include only corporate bonds payable in Canadian dollars with more than 1 year to maturity, we focus only on the bond issues which fall under these categories. The overall amount outstanding for the four companies as of the year-end prior to default is close to \$1.2 billion Canadian. [29] According to the reported maturities of these bonds, 1.38% of this amount represents the short-term bucket, 39.20% represents the long-term bucket, and the largest portion, with 59.42%, is of the mid-term bucket. In light of the domination of the medium-term bonds among the defaulted issues, it is not surprising that in the second sub-period – when the securities of the four real-estate companies carried the A+ and A CBRS ratings – the impact of depressed bond prices is most pronounced on the mid-term AA- and A-rated SCM indices.

For (2), we argue that the long-term and mid-term A-rated indices may still be "contaminated" with bonds carrying a standard call during the last sub-period, which means that the callability term is actually negative and the ratio D^c_a/D^c_m may be less than unity. To support this argument, we estimate regression model (7) at the individual bond level using two A-rated bonds, one carrying a standard call and the other carrying the doomsday call. In order to conduct a clean test of our hypothesis, we look for two such bonds that are similar in every way except for their

[28] During the 01:1990–12:1993 recessionary period, which is also characterized by declining interest rates, we attach a negative sign for $\partial p/\partial r$ for all indices. Kihn (1994) shows that during recessionary periods, which are also characterized by declining interest rates, prices of low-quality bonds will tend to decrease, due to deteriorated credit quality, to a level low enough to eliminate the incentive to call these bonds which arises due to the lower interest rates.

[29] Data are from the *Financial Post* 1992 and 1993 *Corporate Bond Record*.

call provision. That is, optimally, these bonds should have the same issuer and credit rating, and similar maturity and coupon rate. In addition, option-like provisions of many corporate bonds other than callability, make the isolation of the influence of default and call risks on the bonds' effective duration difficult. Thus, our pair of bonds should also exclude such option-like provisions.

The data for the two bonds are gathered from three different sources, which include the Financial Post Corporate Bond Record, the Financial Post Bond Prices, and the data provider, *Bloomberg*. The Financial Post Bond Prices reports the name of the issuer, the coupon rate, maturity date, yield, and price, for the last day of each month in a given year. Data collected from the Financial Post Corporate Bond Record include maturity date, DBRS rating, callability (doomsday call/standard call/noncallable), and all aforementioned option-like provisions. Not surprisingly, finding two bonds issued by the same company with similar maturity, one with a standard call and the other with a doomsday call, is a difficult task. Ensuring that the two bonds do not carry any additional option-like provisions is even harder.

Following an extensive search we focus on two bonds issued by Westcoast Energy Incorporated. The sample covers the 01:1994–12:1997 period. The first bond has a standard call provision, carries a 10.6% coupon rate, and matures on January 15, 2006. Monthly yields for this bond are taken from the Financial Post Bond Prices. The second bond has a doomsday call provision, with a 9.05% coupon rate, and matures on February 4, 2002. Since pricing data for this bond are not available in the Financial Post Bond Prices, monthly yields are taken from *Bloomberg*. Both bonds have an A(low) rating and fall into SCM's mid-term maturity bucket throughout the sample period. In addition, except for callability, neither bonds carries any option-like provisions. [30]

The credit rating for both bonds is stable at A(low) throughout the entire sample period. Thus, it is reasonable to expect that $\partial p/\partial r$ for these bonds is approximately zero. Thus, we are only left with the callability term. For the 9.05% coupon, doomsday bond we expect the callability term to be zero as well, and therefore for this bond we hypothesize $D_{\mathrm{a}}/D_{\mathrm{m}} = 1$. Since the 10.6% coupon bond carries a standard call provision, we expect the callability term to be negative, and therefore, for this bond we hypothesize $D^{\mathrm{c}}_{\mathrm{a}}/D^{\mathrm{c}}_{\mathrm{m}} < 1$.

We apply the Yule–Walker method to estimate regression model (7) for each of the two individual bonds. [31] We report the results of this estimation in Table 3. As expected, the regression slope estimator indicates that $D^{\mathrm{c}}_{\mathrm{a}}/D^{\mathrm{c}}_{\mathrm{m}} < 1$ for Westcoast Energy's standard bond, while we find that $D_{\mathrm{a}}/D_{\mathrm{m}} = 1$ for its doomsday bond. In other words, after controlling for the impact of the default term (i.e., $\partial p/\partial r = 0$),

[30] Duffee (1998) shows that differences in coupon rates have a strong impact on effective duration. Unfortunately, bonds carrying a standard call provision tend to have much higher coupon rates relative to doomsday bonds. To control for this potential bias, in our search we put an emphasis on minimizing coupon differences, at the expense of having maturity differences. Unfortunately, given the data limitations, we could not find a "cleaner" case.

[31] When estimating regression model (7) for the two bonds, we find that OLS residuals follow an AR(1) process.

Table 3
Estimation of the relationship between the default- and call-adjusted duration and its corresponding Macaulay duration for Westcoast Energy Incorporated Bonds during the 01:1994–12:1997 sub-period

Bond	Callability	Slope	Constant	Adj. R^2	q	DW(q)	Iter	SE(δ)	$t(\delta < 1)$	Obs
Westcoast Energy 10.6%	Standard	0.8505 (12.62)	0.0206 (4.07)	0.99	1	2.0460	5	0.0674	2.2181	48
Westcoast Energy 9.05%	Doomsday	0.9141 (12.71)	0.0105 (1.97)	0.78	1	2.0036	6	0.0719	1.1947	48

$$y_{it}^{c} = \gamma + \delta_i r_t + \Psi_{it},$$

where y_{it}^{c} is the continuously compounded yield on bond i at time t. r_t is the continuously compounded yield on a mid-term Government of Canada constant-maturity index, δ_i is the slope coefficient which represents the relationship between the default- and call-adjusted duration of bond i and its Macaulay duration. γ_i is the intercept, and Ψ_{it} is the error term. t-values are in parentheses. In addition to the estimates of the regression parameters and the adjusted R^2, the table reports the following: q gives the degree of the autoregressive process as determined by the AIC, DW(q) reports the Durbin–Watson statistic adjusted for the selected autoregressive order (q), Iter is the number of iterations performed by the Yule–Walker procedure, SE($\delta < 1$) is the standard error of the slope estimator, and $t(\delta < 1)$ is the t-statistic used to test the null hypothesis that the slope coefficient is unity. Obs is the number of monthly observations.

we see that the bond's default- and call-adjusted duration is insignificantly different from its Macaulay duration for Westcoast Energy's economically noncallable doomsday bond. On the other hand, for Westcoast Energy's bond carrying the economically viable standard call, we find that the bond's default- and call-adjusted duration is significantly lower than its Macaulay duration. These results support our argument that the mid-term A-rated indices may be "contaminated" with bonds carrying a standard call during the 01:1994–12:1997 period, which causes the slope estimator to be significantly lower than one.

7. Summary and conclusions

In this paper we develop a unified theoretical framework to derive hypotheses with respect to both default adjustment and call adjustment for risky callable bonds. We show that, while callability will always have the effect of shortening duration, the default-risk adjustment may either lengthen or shorten it. The direction of the impact of risk adjustment depends on the sign of the relationship between changes in the issuer's credit quality and changes in the riskless term structure. We offer a simple methodology for determining the sign of this relationship based on changes in credit-ratings and term-structure movements.

Using the Scotia Capital Markets Canadian corporate bond indices (stratified by rating groups and term buckets) which include a substantial number of callable bonds, we estimate the relationship between the default- and call-adjusted duration and its Macaulay counterpart. Our results support the need for callability adjustment most of the time for callable bond portfolios of all investment-grade ratings. Even when the default adjustment is expected to lengthen duration, we still find that the default- and call-adjusted duration is lower than its Macaulay counterpart.

The unique doomsday call provision attached to most Canadian corporate bonds issued starting in 1987, makes them economically noncallable and allows us to isolate the impact of callability from our analysis. In particular, the risk adjustment for the long-term bond indices tends to become less important as we move from our first to the last sub-period. We believe that this is due to the continuously changing composition of these indices: A trend that reflects the inclusion of more and more of the economically noncallable bonds in our indices, starting 1987.

Although our default adjustment does not gain much support from the data, we do find some evidence suggesting that the skyrocketing (real-estate crisis related) default rates of AA- and A-rated bonds during the 1991–1993 period may have caused investors to price these bonds similarly to high-yield bonds. We conclude that the depressed prices for medium-term AA- and A-rated bonds during this period eliminate call risk. This explains our observation that the risk-adjusted duration is at levels comparable to those of Macaulay duration for bonds of these ratings.

The results for our investment-grade bond indices also support Kihn's (1994) hypothesis for high-yield bonds and the implications of Acharya and Carpenter's (2000) bond pricing model. Kihn claims that during a recessionary period characterized by declining interest rates (such as our second sub-period), bond prices will tend to decrease, due to deteriorated credit quality, to a level low enough to eliminate the incentive to call these bonds arising from lower interest rates. Comparing our results for the second sub-period with the results for the other two periods, it appears that the call adjustment is less important – although still significant – during the second sub-period. This result is predicted by Acharya and Carpenter's (2000) pricing model for callable defaultable bonds.

Our conclusion is that portfolio managers holding callable risky bonds and using duration either as an immunization tool or in rate anticipation strategies, must adjust their duration measure for callability. We cannot however, reach a similar conclusion with respect to the impact of the default adjustment. These results indicate that the call risk of a typical investment-grade bond poses a greater danger to a successful immunization strategy than does default risk.

Further, our results support the main conclusion in Duffee's (1998) yield-spread study: Failing to control for the callability of bonds may lead to spurious conclusions about the impact of default risk on the price elasticity of corporate bonds with respect to the riskless term structure. In particular, Fons (1990) does not consider callability an important factor in measuring this risk-adjusted price elasticity. Thus, he concludes that default risk is the reason for his results wherein the empirical risk-adjusted price elasticity is significantly lower than the empirical Macaulay duration of US corporate bond indices of all ratings. The conclusions of our study clearly support the view that Fons' results likely proxy for the impact of callability rather than measure the impact of default.

The composition of Canadian corporate bond indices is continuously changing, reflecting the inclusion of more and more new bonds carrying the economically noncallable doomsday call provision since 1987. If Canadian corporations continue issuing bonds with the doomsday call for the next several years, Canadian corporate bond indices of all maturities will be considered economically noncallable. This trend will provide researchers with a reliable data set of Canadian corporate bond indices, which facilitates the study of the impact of default risk on both yield spreads and the risk-adjusted duration. Research utilizing this data will be subject to neither Duffee's (1998) nor our criticism of the use of ordinary corporate bond indices to study the impact of default risk.

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